

Dualität

Vektorraum

$$V$$

15.1

Dualraum

$$V^*$$

Linearformen
 $f: V \rightarrow \mathbb{K}$

Basis:

$$B = (v_1, \dots, v_n)$$

15.2

duale Basis

$$B^* = (v_1^*, \dots, v_n^*)$$

$$v_i^*(v_j) := \delta_{ij} = \begin{cases} 1 & (i=j) \\ 0 & (i \neq j) \end{cases}$$

Galoisverbindung

$$X \subseteq V$$

$$Y^\circ := \{v \in V \mid \forall f \in Y: f(v) = 0\}$$

Annulator



Annulator

15.13
(15.12)



$$X^\circ := \{f \in V^* \mid \forall v \in X: f(v) = 0\}$$

$$Y \subseteq V^*$$

$X \subseteq V$ Galois-Hüllen $Y \subseteq V^*$
 $\dim X = k \iff \dim X^\circ = n - k$
 $\dim Y^\circ = n - k \iff \dim Y = k$

lin. Abb

$$\varphi: V \rightarrow W$$

15.14

duale Abb.

$$\varphi^\circ: W^* \rightarrow V^*$$

$$\varphi^\circ(g) := g \circ \varphi$$

