Existence of solutions to a phase–field model of dynamic fracture with a crack–dependent dissipation

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Abstract

In this talk I present some phase–field models of dynamic brittle fracture, based on a suitable adaptation of Griffith's criterion. In these models, which rely on the Ambrosio–Tortorelli's approximation, the $(d-1)$–dimensional set that represents the crack is replaced by a function $v \in [0,1]$, called phase–field, which takes a value near 0 in a small neighborhood of the crack set, and a value near 1 far from it.

By using a time discretization scheme, I prove the existence of a solution for a particular model which avoids viscoelastic terms on the displacement, and takes into account dissipative effects due to the crack tips speed. Finally, under suitable assumptions on this dissipative term, I show that the evolution satisfies a dynamic energy–dissipation balance, according to Griffith's criterion.

Elek Csobo

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Orbital stability of a Klein-Gordon equation with Dirac delta potentials

We study the orbital stability of standing wave solutions of a one-dimensional nonlinear Klein-Gordon equation with Dirac potentials

$$\begin{cases} u_{tt} - u_{xx} + m^2 u + \gamma\delta(x)u + i\alpha\delta(x)u_t - |u|^{p-1}u = 0, \\ u(t,x) \to 0, \quad \text{as} \quad |x| \to \infty, \\ (u(t), \partial_t u(t))|_{t=0} = (u_0, u_1), \end{cases}$$

where $\delta(x)$ is the Dirac mass at $x = 0$. The general theory to study orbital stability of Hamiltonian systems was initiated by the seminal papers of Grillakis, Shatah, and Strauss [1, 2], newly revisited by De Bièvre *et. al.* in [3]. I present the Hamiltonian structure of the above system and the orbital stability properties of the standing wave solutions of the equation. A major difficulty is to determine the number of negative eigenvalues of the linearized operator around the stationary solution, which we overcome by a perturbation argument. This is a joint work with Masahito Ohta, François Genoud and Julien Royer.

References

[1] M. Grillakis, J. Shatah, W. Strauss, Stability theory of solitary waves in the presence of symmetry, I. *J. of Funct. Anal.* **74** (1987), no. 1, 160–197.

[2] M. Grillakis, J. Shatah, W. Strauss, Stability theory of solitary waves in the presence of symmetry, II. *J. of Funct. Anal.* **94** (1990), no. 2, 308–348.

[3] S. De Bièvre, F. Genoud, S. Rota-Nodari, Orbital stability: analysis meets geometry, in *Nonlinear optical and atomic systems,* **2146** (2015), 147–273.

Wellposedness of deterministic and stochastic nonlinear Dispersive Equations

by Tobias Friesel

This talk consists of two parts:
In the first part I will give a general introduction to the local and global Wellposedness of nonlinear Dispersive Equations using the example of the Schrödinger Equation.

$$(i\partial_t + \triangle)u = \pm|u|^{p-1}u \qquad \textit{in } \mathbf{R}^n \times (0,T) \tag{1}$$
$$u = u_0 \qquad \textit{on } \mathbf{R}^n \times \{0\} \tag{2}$$

In the second part we will consider nonlinear Dispersive Equations with additive noise and in particular the Schrödinger Equation.

$$(i\partial_t + \triangle)u = \pm :|u|^{2k}u: + \xi \qquad \textit{in } \mathbf{T}^n \times (0,T) \tag{3}$$
$$u = u_0 \qquad \textit{on } \mathbf{T}^n \times \{0\} \tag{4}$$

Over the recent years there has been an ongoing interest in studying this kind of stochastic nonlinear PDEs and thanks to Martin Hairer's theory of regularity structures we have a good understanding for parabolic PDEs. By comparison Dispersive Equations like the Wave Equation and Schrödinger Equation are much less well understood. However a recent paper [GKO18] [1] proved the Wellposedness results for the 2-dimensional periodic Wave Equation with additive space-time-white-noise. The approach used in this paper can also be generalized to larger classes of Dispersive Equations and Noise.

I will give a brief overview of the core aspects and difficulties of studying this kind of problem and the power and limitations of available approaches. We will also see why Wellposedness results for the Schrödinger Equation are currently only possible, if we consider a regularized noise and how we can modify the setting to be able to gain more powerful results.

[1] Massimiliano Gubinelli, Herbert Koch and Tadahiro Oh
Renormalization of the two-dimensional stochastic nonlinear wave equation
Trans. A.M.S 370, 2018

Raffaele Grande

Horizontal mean curvature and the mean field equation in $\mathbb{H}^1$

The *horizontal mean curvature flow* (HMCF) and the mean field equation in sub-Riemmanian setting are a partial differential equations which have applications in biomathematics and neurogeometry. Unfortunately in these spaces there is a very weak regularity (Hormander's condition) then the associated PDEs are very degenerate. One possible approach is to complete the underlying sub-Riemmanian geometry to a Riemmanian geometry depending upon a small parameter $\varepsilon > 0$, then the associated PDEs became much more regular. Thus we can derive informations on the starting degenerate problem as $\varepsilon \to 0$. We study the case of HMCF in *Heisenberg group* with a stochastic approach via a Riemmanian approximation and in the second part we will study the mean field equation in the Heisenberg group.

A CONTINUITY RESULT FOR THE TRACE OPERATOR IN THE CONTEXT OF SPECIAL FUNCTIONS WITH BOUNDED VARIATION

EMANUELE TASSO

The space $SBV(\Omega)$ of special functions with bounded variation, and $GSBV(\Omega)$ of generalised special functions with bounded variation, have been introduced to study the so called *free discontinuity problems*. In these spaces it is possible to define a trace operator, whose definition coincides with the usual one, when u is a Sobolev function. Unfortunately, due to the fact that a sequence in $(G)SBV(\Omega)$ may have jump sets getting infinitesimally close to the boundary of Ω, the trace operator is not continuous. This lack of continuity leads for example to free discontinuity problems with no solution.

In this talk I present a possible way to overcome this problem, by restricting our attention on a smaller class of functions. Given Γ an $(n-1)$-dimensional set, we consider the space $(G)SBV(\Omega;\Gamma)$ of functions in $(G)SBV(\Omega)$, whose jump sets are contained in Γ. In these spaces it is possible to introduce a suitable weight function on the $\mathcal{H}^{n-1}$ measure of $\partial\Omega$, to obtain some continuity results for the trace operator. Finally, I show an application of this result, to a suitable class of free discontinuity problems with boundary conditions.