Applied Analysis Day
TU Dresden, 28 – 29 June 2018

Estimating critical cavitation loads in nonlinear elasticity
Jonathan Bevan (Surrey)

A well-known approach to modelling elastic materials postulates that, subject to a suitable boundary condition, the material will deform so as to minimise its elastic stored energy. It is well know that it can be energetically favourable for such a deformation to cavitate, i.e. open a 'hole' inside the material, which corresponds to a discontinuity in the mapping. One way to rule this out is to prove directly that a global minimiser is an affine map, thereby guaranteeing that phenomena like cavitation do not lower the energy. This amounts to a study of quasiconvexity, a fundamental condition discovered by C. B. Morrey in the 1950s, in the setting of nonlinear elasticity. The talk will show how quasiconvexity, rigidity estimates from the calculus of variations and singular values of gradient matrices combine to give sufficient conditions for global minimisers to be affine maps of suitable stored-energy functionals.

Stochastic homogenisation of high-contrast media
Mikhail Cherdantsev (Cardiff)

Using a suitable stochastic version of the compactness argument of V. V. Zhikov, we develop a probabilistic framework for the analysis of heterogeneous media with high contrast. We show that an appropriately defined multiscale limit of the field in the original medium satisfies a system of equations corresponding to the coupled "macroscopic" and "microscopic" components of the field, giving rise to an analogue of the "Zhikov function", which represents the effective dispersion of the medium. We demonstrate that, under some lenient conditions within the new framework, the spectra of the original problems converge to the spectrum of their homogenisation limit.

Dispersive effective behaviour of high-contrast periodic media
Kirill Cherednichenko (Bath)

I will discuss my recent work with Y. Ershova and A. Kiselev, demonstrating that spectral problems for quantum graphs with rapidly oscillating high-contrast weights are asymptotically equivalent to "homogenised" models with energy-dependent interface conditions. We show that these asymptotically equivalent models are directly related (in the sense of Schur-Frobenius duality) to models for time-dispersive media, which in the time domain involve memory, and we characterise the corresponding time convolution kernels explicitly.

Flexibility and rigidity of isometric embeddings
Dominik Inauen (Zürich)

I will talk about the problem of embedding a Riemannian manifold isometrically (i.e. preserving distances) into Euclidean space. The focus will lie on the interesting difference in behavior of these maps at low regularity (for example: C^1 isometric embeddings of positively curved surfaces are totally flexible, whereas embeddings which are C^2 are rigid), which also has a connection to the famous conjecture of Onsager in fluid dynamics.

Homogenization and thin film asymptotics for slender structures in nonlinear elasticity
Matthäus Pawelczyk (Dresden)

The study of thin elastic materials is often conducted in the setting of calculus of variations, where the behavior of (almost) minimizers is studied. In this setting we extend the previously known simultaneous dimension reduction and homogenization of plate to stochastic inhomogeneities. For this we develop a 'mixed Helmholtz decomposition'.
Furthermore, we show that for rods such models, are not only related by minimizers. Under suitable growth conditions on the stress, we show that also stationary points converge appropriately. Depending on the chosen notion of stationarity, hence also a natural growth condition for the stress, such results can be derived for different scaling regimes.
This talk is a slight variant of the defense of my PhD thesis given the previous day.

Derivation of continuum models for epitaxially-strained films and regularity properties

Paolo Piovano (Wien)

Variational models for thin films deposited on substrates are derived by relaxation from the sharp-interface model and by Γ-convergence from the transition-layer model, as well as by means of a discrete-to-continuum analysis from atomistic models.

Because of the mismatch between the film and the substrate lattices at their crystalline equilibrium the thin-film material is epitaxially strained, and the resulting limiting continuum model includes a bulk elastic term besides the surface energy related to the free film profile. Furthermore, different elastic properties between the film and the substrate are allowed and the surface tension distinguishes among all three involved interfaces, i.e., film/gas, substrate/gas, and film/substrate.

The regularity of energy-minimal film profiles is then studied by establishing the internal-ball condition and by adopting some arguments from transmission problems. The results relate to both the Stranski-Krastanow and the Volmer-Weber modes. Moreover, geometrical conditions are provided for the optimal wetting angle, i.e., the angle formed at the contact points between the film and the substrate. In particular, the Young-Dupré law is shown to hold, yielding what appears to be the first analytical validation of such law in the context of Continuum Mechanics.

Shape Design of a Polymer Microstructure for Bones

Stefan Simon (Bonn)

We consider a shape optimization problem related to the design of scaffolds for bone tissue engineering. One possibility for such bone scaffolds are porous structures made from biologically degradable polymers. Our aim is to compute a suitable microstructure of this polymer, which should provide adequate mechanical stability during the regeneration time. With the intension that bone first grows into the void material, we also require the complementary set filled with bone to be stable. This can be formulated as a shape optimization problem, where we minimize the maximum value of both compliances. We present a numerical scheme via a phase-field approach.

Analysis for the discrete approximation of damage and fracture

Marita Thomas (Berlin)

This presentation deals with techniques for the spatial and temporal discretization of damage models featuring a gradient regularization. Different challenges in dependence of the choice of the gradient term and the damage evolution law as well as related convergence results are discussed. In this context, the talk also addresses a phase-field fracture model at finite strains, which takes into account the anisotropy of damage by applying an anisotropic split and the modified invariants of the right Cauchy-Green strain tensor.

Operator norm long-wave asymptotics for elastic plates with rapidly oscillating periodic properties

Igor Velčić (Zagreb)