



PYTHON FOR ENGINEERS

PYTHONKURS FÜR INGENIEUR:INNEN

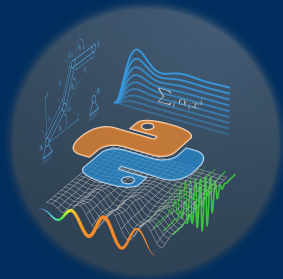
Symbolic Computation with Sympy

Symbolisch Rechnen mit Sympy

Slides: Carsten Knoll

<https://tu-dresden.de/pythonkurs>
<https://python-fuer-ingenieure.de>

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- goal: overview of computer algebra possibilities (`import sympy`).
- structure:
 - introduction
 - calculating
 - substituting
 - important functions & data types
 - evaluate formulas numerically

- Python library for symbolic computations ('calculations with letters')
- backend of a **computer algebra system** (CAS)
- possible frontends: own Python script, IPython shell, Jupyter notebook (in browser)
- advantage over other products: CAS-functionality inside a real programming language

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- ways to import the package or objects from it:
 1. `import sympy as sp`
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- `sp.symbols, sp.Symbol` : create symbols (\neq variables)
- `sp.sin(x), sp.cos(2*pi*t), sp.exp(x), ...` : mathematical functions
- `sp.Function('f')(x)` : create and evaluate custom functions
- `sp.diff(<expr>, <var>)` or `<expr>.diff(<var>)` : taking derivatives
- `sp.simplify(<expr>)` : simplification
- `<expr>.expand()` : expansion of brackets ($a(x + y) \rightarrow ax + ay$)
- `<expr>.subs(...)` : substitution
- `sp.pprint(<expr>)` : “pretty printing”

Listing: sympy1.py

```
import sympy as sp
x = sp.Symbol("x")
a, b, c = sp.symbols("a b c") # different ways to create symbols

z = a*b*x*b + b**2*a*x - c*b*(2*a/c*x*b-1/(b**2))
print(z) # -> -b*c*(-1/(2*b) + 2*a*b*x/c) + 2*a*x*b**2
print(z.expand()) # -> c/2 (multiply out)

# apply functions:
y = sp.sin(x)*sp.exp(3*x)*sp.sqrt(a)
print(y) # -> a**(1/2)*exp(x)*sin(x)

# define custom function
f1 = sp.Function("f") # -> sympy.core.function.f (not evaluated)
g1 = sp.Function("g")(x) # -> g(x) (function evaluated at x)

# taking derivatives
print(y.diff(x)) # -> 3*sqrt(a)*exp(3*x)*sin(x) + sqrt(a)*exp(3*x)*cos(x)
print(g1.diff(x)) # -> Derivative(g(x), x)

# simplification (example):
y = sp.sin(x)**2+sp.cos(x)**2
print(y == 1) # -> False
print(sp.simplify(y)) # -> 1
print(sp.simplify(y) == 1) # -> True
```

- comparable to `<str>.replace(old, new)`
- useful for: manual simplifications, (partial) function evaluations, coordinate transformations.

```
term1 = a*b*sp.exp(c*x)
term2 = term1.subs(a, 1/b)
print(term2) # -> exp(c*x)
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- call options: 1. two arguments, 2. list with 2-tuples, 3. dict
 1. `<expr>.subs(old, new)`
 2. `<expr>.subs([(old1, new1), (old2, new2), ...])`
 3. `<expr>.subs({old1: new1, old2: new2, ...})`
 - option 2: order of list → substitution order
 - relevant when substituting derivatives (see [example notebook](#))
- important: `.subs(...)` returns new expression (original expr. remains unchanged)

More Important Functions, Methods, Types

- `A = sp.Matrix([[x1, a*x2], [c*x3, sp.sin(x1)]])` : create a matrix
- `<mtrx>.jacobian(xx)` : Jacobian matrix of a vector (matrix of partial derivatives)
- `sp.solve(x**2 + x - a, x)` : solve (systems of) equations
- `<expr>.atoms()`, `<expr>.atoms(sp.sin)` : “atoms” (of a certain type)
- `<expr>.args` : arguments of the respective class (summands, factors, ...)
- `sp.sympify(...)` : adapt data type (plain Python \rightarrow Sympy)
- `sp.integrate(<expr>, <var>)` : symbolic integration (anti derivative)
- `sp.series(...)` : Taylor series expansion of expression $\left(\text{like } e^x \Big|_{x=0} = \sum_{k=0}^n \frac{1}{k!} x^k\right)$
- `sp.limit(<var>, <value>)` : limit $\left(\text{like } \lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1\right)$
- `<expr>.as_num_denom()` : decomposition into 2-tuple: (*numerator, denominator*)
- `sp.poly(x**7+a*x**3+b*x+c, gens=[x], domain="EX")` : Polynomial
- `sp.Piecewise(...)` : piecewise defined function

\rightarrow see docs (and docstrings) for more info (or just try things out)

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- better (in terms of speed): `lambdify` (origin of the name: Python's `lambda` functions)
- creates a Python function that you can then call with the arguments

```
f = a*sp.sin(b*x)
df_xa = f.diff(x)

# create the function
df_xa_fnc = sp.lambdify((a, b, x), df_xa, modules='numpy')

# call (= evaluate) the function
print( f_xa_fnc(1.2, 0.5, 3.14) )
```

Another trick: let sympy expressions be nicely rendered by $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X} \Rightarrow$ readability ↑.

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In [7]: from sympy.interactive import printing
printing.init_printing()

%load_ext ipydx.displaytools
```

```
In [8]: # same code with special-comments ('##:')

x = sp.Symbol("x")
a, b, c, z = sp.symbols("a b c z") # create several symbols at once

some_formula = a*b*x*b + b**2*a*x - c*b*(2*a/c*x*b-1/(b**2)) ##:

# some calculus
y = sp.sin(x)*sp.exp(3*x)*sp.sqrt(a) ##:

# derive
yd = y.diff(x) ##:
```

$$\text{some_formula} := 2ab^2x - bc \left(\frac{2a}{c}bx - \frac{1}{2b} \right)$$

$$y := \sqrt{ae^{3x}} \sin(x)$$

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[See Example Notebook](#)

[../notebooks/course09-sympy-demo.html](#)

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- docstrings of the respective objects (in Jupyter: e.g. `sp.solve?`)
- <http://docs.sympy.org/latest/tutorial/index.html>
- module reference (e.g.: `solve` -function)
- <http://docs.sympy.org/latest/tutorial/gotchas.html> (pitfalls)

- symbolic calculations
- substitution
- important functions / data types
- numerical evaluation of functions