# Ma 2 Taylor-Formel mehrdimensional

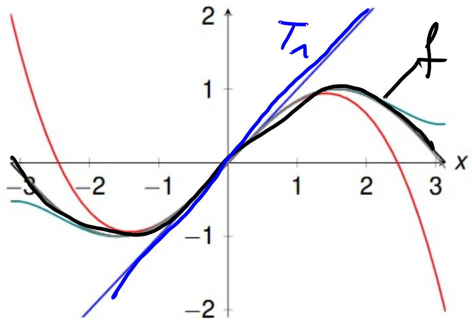
**Ü2 Aufgabe 18.9. a,d**

Mit Hilfe der Taylorschen Formel approximiere man die durch $z = f(x,y)$ gegebene Fläche an der Stelle $P(x_0, y_0)$ durch eine Fläche 2. Ordnung.

a) $z = y\ln(y-3x)$, $\underbrace{P(0;1)}_{x_0, y_0}$,

Wdh VL

Taylor 1-dim. $f(x) = \frac{f(x_0)}{0!} + \frac{f'(x_0)\cdot(x-x_0)}{1!} + \frac{f''(x_0)\cdot(x-x_0)^2}{2!} + \dots$ $T_1 \rightarrow$ $T_2 \rightarrow$




Taylor mehrdim. $T_2(\underline{x}) = f(\underline{x}^0) + \nabla f(\underline{x}^0)^T(\underline{x}-\underline{x}^0) + \frac{1}{2}(\underline{x}-\underline{x}^0)^T H_f(\underline{x}^0)\cdot(\underline{x}-\underline{x}^0)$ $T_1 \rightarrow$

$f: R^n \rightarrow R$

Taylor 2 dim.

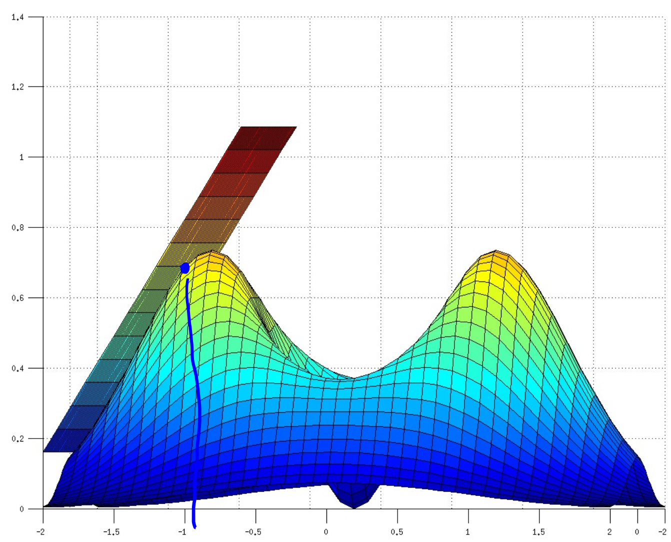
$f: R^2 \rightarrow R$

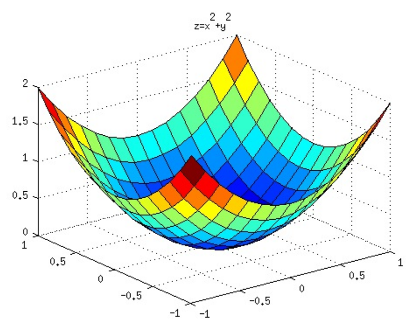
$$\cancel{f(x,y) \approx}\; T_2(x,y) = f(x_0,y_0) + \frac{1}{1!}\nabla f(x_0,y_0)\begin{bmatrix} x-x_0 \\ y-y_0\end{bmatrix} + \frac{1}{2!}\begin{bmatrix} x-x_0 \\ y-y_0\end{bmatrix}^T \begin{bmatrix} f_{xx}(x_0,y_0) & f_{xy}(x_0,y_0) \\ f_{yx}(x_0,y_0) & f_{yy}(x_0,y_0)\end{bmatrix}\begin{bmatrix} x-x_0 \\ y-y_0\end{bmatrix}$$

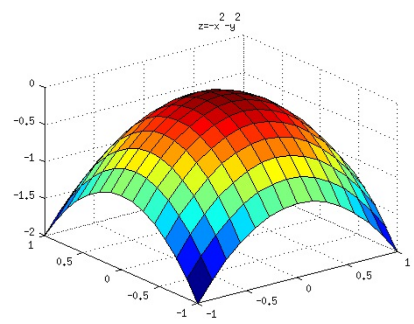
$c$ $\quad a, b$ $\quad T_1 \rightarrow$ $\quad T_2 \rightarrow$

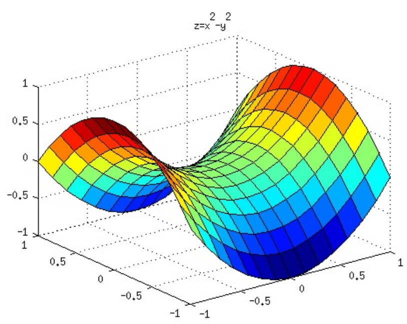
$$\begin{pmatrix} d & g \\ g & e \end{pmatrix}$$

$$d(x-x_0)^2 + 2g(x-x_0)(y-y_0) + e(y-y_0)^2$$







$z = f(x,y) = y\ln(y-3x) \quad\Rightarrow\quad f(0,1) = 0$

$f_x(x,y) = \frac{-3y}{y-3x} \quad\Rightarrow\quad f_x(0,1) = -3$

$f_y(x,y) = \ln(y-3x) + \frac{y}{y-3x} \quad\Rightarrow\quad f_y(0,1) = 1$

$f_{xx}(x,y) = \frac{-9y}{(y-3x)^2} \quad\Rightarrow\quad f_{xx}(0,1) = -9$

$f_{xy}(x,y) = f_{yx} = \frac{-3(y-3x)+3y}{(y-3x)^2} = \frac{9x}{(y-3x)^2} \quad\Rightarrow\quad f_{xy}(0,1) = 0$

$f_{yy}(x,y) = \frac{1}{y-3x} - \frac{3x}{(y-3x)^2} \quad\Rightarrow\quad f_{yy}(0,1) = 1$

$$T_2(x,y) = 0 + (-3, 1)\begin{pmatrix} x \\ y-1 \end{pmatrix} + \frac{1}{2}(x, y-1)\begin{pmatrix} -9 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y-1\end{pmatrix} = \underline{\underline{-3x + y - 1 - \frac{9}{2}x^2 + \frac{(y-1)^2}{2}}}$$