STATISTICAL PRINCIPALS AND COMPUTATIONAL METHODS.
2. SEMINAR – ENERGY MINIMIZATION, SEARCH TECHNIQUES

Exercise 1. Consider one "elementary step" of the ICM-Algorithm, i.e. the choice of the best label in a node given the fixed rest:

$$y_i = \arg\min_{k \in K} \Big[\psi_i(k) + \sum_{ij \in \mathscr{E}} \psi_{ij}(k, y_j) \Big].$$

The time complexity of this step is $O(m|K|)$ in general, where m is the number of nodes that are linked with i by an edge and $|K|$ is the number of labels.

How to improve the time complexity, if the functions ψ_{ij} are:

a) the Potts model $\psi_{ij}(k, k') = a_{ij} \cdot \delta(k \neq k')$

b) quadratic distance between labels $\psi_{ij}(k, k') = a_{ij} \cdot (k - k')^2$

c) hard restrictions $\psi_{ij}(k, k') = \begin{cases} 0 & \text{if } |k - k'| < \triangle \\ \infty & \text{otherwise} \end{cases}$

d) a modified version of **c)** $\psi_{ij}(k, k') = \begin{cases} a & \text{if } |k - k'| < \triangle \\ b & \text{otherwise} \end{cases}$

Exercise 2. Re-parameterize a binary MinSum task (i.e. $K = \{0, 1\}$)

$$y^* = \arg\min_{y} \Big[\sum_i \psi_i(y_i) + \sum_{ij} \psi_{ij}(y_i, y_j) \Big].$$

so that it can be written in the form

$$y^* = \arg\min_{y} \Big[\sum_i \psi_i \cdot y_i + \sum_{ij} \psi_{ij} \cdot y_i \cdot (1 - y_j) \Big].$$

with nodes- and edge-specific numbers ψ_i and ψ_{ij}.

Exercise 3. Consider a binary MinSum task given in the canonical form

$$y^* = \arg\min_{y} \Big[\sum_i \psi_i(y_i) + \sum_{ij} \beta_{ij} \cdot \delta(y_i \neq y_j) \Big].$$

Numbers β_{ij} are *non-negative* and functions ψ_i are *arbitrary*. Transform the task into a MinCut one with *only non-negative* edge costs.

Note: in the "standard" transformation MinSum→MinCut some edge costs of the latter are just the values of the corresponding ψ_i (i.e. c_{si} and c_{it} – see the lecture, slide 14).

Is it possible to modify the unary potentials ψ_i of the original MinSum problem so that (i) all their values are non-negative and (ii) the modified MinSum task is equivalent (in some sense) to the original one?

Exercise 4. Let pairwise functions ψ_{ij} of a MinSum problem are defined as follows. Consider a graph, whose nodes correspond to labels. Edges of this graph have costs. The value of ψ_{ij} for a label pair (k,k') is the cost of the shortest path (sum of the edge-costs along the path) from the node k to the node k' in the graph.

Prove that the auxiliary tasks for α-expansion are submodular.

Hint: Use the fact presented in slide 21 of the lecture.

Exercise 5. Let a submodular binary MinSum problem be given. We start from a random labeling and perform α-expansions until convergence.

Prove that this procedure finds the globally optimal solution.